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Rarefactive ion-acoustic electrostatic solitary structures in nonthermal plasmas

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Abstract. An investigation has been made of ion-acoustic solitary waves in an unmagnetized nonthermal plasma whose constituents are an inertial ion fluid and nonthermally distributed electrons. The properties of stationary solitary structures are briefly studied by the pseudo-potential approach, which is valid for arbitrary amplitude waves, and by the reductive perturbation method which is valid for small but finite amplitude limit. The time evolution of both compressive and rarefactive solitary waves, which are found to coexist in this nonthermal plasma model, is also examined by solving numerically the full set of fluid equations. The temporal behaviour of positive (compressive) solitary waves is found to be typical, *i.e.*, the positive initial disturbance breaks up into a series of solitary waves with the largest in front. However, the behaviour of negative (rarefactive) solitary waves is quite different. These waves appear to be unstable and produce positive solitary waves at a later time. The relevancy of this investigation to observations in the magnetosphere of density depressions is briefly pointed out.

PACS. 52.35.Fp Electrostatic waves and oscillations (e.g., ion-acoustic waves) – 52.35.Sb Solitons; BGK modes – 52.35.Mw Nonlinear waves and nonlinear wave propagation (including parametric effects, mode coupling, ponderomotive effects, etc.)

1 Introduction

Nonlinear propagation of electrostatic disturbances in space and laboratory plasmas has received a considerable attention and been extensively studied in last few years [1–6]. It has been found both theoretically and experimentally that only positive (compressive) solitary waves (solitary waves with positive potential or with density hump) exit in two component plasmas, but the system does not support any negative (rarefactive) solitary waves (solitary waves with negative potential or with density dip). Observations made by the Viking spacecraft [7] and Freja satellite [8] have found electrostatic solitary structures in the magnetosphere with density depressions. Sometimes these are associated with small scale lower hybrid turbulence, in which case they can be explained as cavities generated by the ponderomotive force of the lower hybrid waves. However, sometimes they have no turbulence associated with them. Recently, motivated by the latter class of events, Cairns *et al.* [9,10] have considered a nonthermal plasma model and shown that the presence of a nonthermal distribution of electrons may change the nature of ion sound solitary structures and allow the existence of structures very like those observed by the Freja and Viking satellites [7,8]. The present study has considered the nonthermal plasma model of Cairns *et al.* [9,10] and mainly extended these recent investigations [9,10] to time evolution of these electrostatic solitary structures in nonthermal plasmas by solving numerically the full set of fluid equations.

The manuscript is organized as follows. The basic equations governing the nonthermal plasma model under investigation are given in Section 2. The arbitrary amplitude compressive and rarefactive ion-acoustic solitary structures are briefly studied by pseudo-potential approach in Section 3. The small but finite amplitude limit is then considered in Section 4. The time evolution of these solitary structures are investigated in Section 5. Finally, a brief discussion is presented in Section 6.

2 Governing equations

We consider a plasma system consisting of an inertial ion fluid and nonthermally distributed electrons. The basic system of equations governing the ion dynamics in this plasma system is given by [9,10]

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nu) = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}\left(\frac{1}{2}u^2 + \varphi\right) = 0, \tag{2}$$

$$\frac{\partial^2 \varphi}{\partial x^2} = \left(1 - \beta\varphi + \beta\varphi^2\right) \mathrm{e}^{\varphi} - n \tag{3}$$

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where n is the ion density normalized to the unperturbed ion density n_0; u is the ion fluid velocity normalized to the ion-acoustic speed $C_s = (K_B T_e/m)^{1/2}$ with T_e, K_B and m being the electron temperature, Boltzmann constant and ion mass, respectively; φ is the electrostatic potential normalized to $K_B T_e/e$ with e being the electronic charge; $\beta = 4\alpha/(1+3\alpha)$ with α being the parameter determining the number of nonthermal electrons present in our nonthermal plasma model [9–11]; the space variable (x) is normalized to the Debye length $\lambda_D = (K_B T_e/4\pi n_0 e^2)^{1/2}$ and the time variable (t) is normalized to the ion plasma period $\omega_p^{-1} = (m/4\pi n_0 e^2)^{1/2}$.

3 Arbitrary amplitude solitary structures

To study time-independent solitary structures, we make all the dependent variables depend only on a single variable $\xi = x - Mt$ (where ξ is again normalized to λ_D and M is the Mach number, soliton velocity/C_s), use the steady state condition, impose the appropriate boundary conditions (namely $n \to 1$, $u \to 0$, $\varphi \to 0$ and $d\varphi/d\xi \to 0$ at $\xi \to \pm\infty$) and reduce our basic equations (1–3), to the "energy integral" [9,10]:

$$\frac{1}{2}\left(\frac{d\varphi}{d\xi}\right)^2 + V(\varphi) = 0, \tag{4}$$

where the Sagdeev potential [12] for our purposes reads

$$V(\varphi) = 1 + 3\beta + M^2 - M^2(1 - 2\varphi/M^2)^{1/2} - [1 + 3\beta(1-\varphi) + \beta\varphi^2]e^{\varphi}. \tag{5}$$

It is clear from (5) that $V(\varphi) = 0$ and $dV(\varphi)/d\varphi = 0$ at $\varphi = 0$. Therefore, solitary wave solutions of (4) exist if (i) $\left(d^2V/d\varphi^2\right)_{\varphi=0} < 0$, so that the fixed point at the origin is unstable and (ii) $\left(d^3V/d\varphi^3\right)_{\varphi=0} > (<)\, 0$ for compressive (rarefactive) solitary waves. The nature of these solitary waves, whose amplitude tends to zero as the Mach number M tends to its critical value, can be found by expanding the Sagdeev potential to third order in a Taylor series in φ. The critical Mach number is that which corresponds to the vanishing of the quadratic term. At the same time, if the cubic term is negative, there is a potential well on the negative side and if the cubic term is positive, there is a potential well on the positive side. Therefore, by expanding the Sagdeev potential $V(\varphi)$, given by (5), around the origin the critical Mach number, at which the second derivative changes sign, can be found as $M_c = \sqrt{(1+3\alpha)/(1-\alpha)}$ and at this critical value of M the third derivative is negative, *i.e.*, both the compressive and rarefactive solitary waves exist if $\alpha > (\sqrt{3}-1)/(3+\sqrt{3}) \simeq 0.155$. The upper limit of M can be found by the condition $V(\varphi_c) \geq 0$, where $\varphi_c = M^2/2$ is the maximum value of φ for which the ion density n is real. Thus, on using (5) this upper limit of M, for $\alpha = 0.2$, can be found as 1.6. Clearly, in nonthermal plasma with $\alpha = 0.2$, finite amplitude ion-acoustic solitary waves exist for $1 < M < 1.6$. The nature of these compressive and rarefactive solitary waves are discussed in more detail by Cairns *et al.* [9,10].

4 Small amplitude limit: K-dV solitons

To study small but finite amplitude ion-acoustic solitary waves in our plasma model, we construct a weakly nonlinear theory of the ion-acoustic waves which leads to the scaling of the independent variables through the stretched coordinates [13]

$$\xi = \epsilon^{1/2}(x - v_0 t), \qquad \tau = \epsilon^{3/2} t, \tag{6}$$

where ϵ is a small parameter measuring the weakness of the dispersion, v_0 is the wave phase velocity normalized to C_s. We can expand the perturbed quantities n, u_z, and φ about their equilibrium values in power of ϵ as [13]

$$n = 1 + \epsilon n^{(1)} + \epsilon^2 n^{(2)} + \cdots$$
$$u_x = 0 + \epsilon u_x^{(1)} + \epsilon^2 u_x^{(2)} + \cdots$$
$$\varphi = 0 + \epsilon\varphi^{(1)} + \epsilon^2\varphi^{(2)} + \cdots \tag{7}$$

Now, using equations (6, 7) in equations (1–3) and following reference [13] one can obtain a nonlinear equation, known as Korteweg-de Vries (K-dV) equation, as

$$\frac{\partial\varphi^{(1)}}{\partial\tau} + a\varphi^{(1)}\frac{\partial\varphi^{(1)}}{\partial\xi} + b\frac{\partial^3\varphi^{(1)}}{\partial\xi^3} = 0, \tag{8}$$

where a and b are given by

$$a = \frac{3}{2}S_\alpha\sqrt{(1-\beta)}, \qquad b = \frac{1}{2}\frac{1}{(1-\beta)^{3/2}}, \qquad S_\alpha = 1 - \frac{1}{3(1-\beta)^2}\,. \tag{9}$$

The steady state solution of this K-dV equation is obtained by transforming the independent variables ξ and τ to $\eta = \xi - u_0\tau$ and $\tau = \tau$, where u_0 is a constant velocity normalized to C_s, and imposing the appropriate boundary conditions, *viz.*, $\varphi \to 0$, $d\varphi^{(1)}/d\eta \to 0$, $d^2\varphi^{(1)}/d\eta^2 \to 0$ at $\eta \to \pm\infty$. Thus, one can express the steady state solution of this K-dV equation as

$$\varphi^{(1)} = \varphi_m^{(1)}\,\mathrm{sech}^2[(\xi - u_0\tau)/\delta], \tag{10}$$

where the amplitude $\varphi_m^{(1)}$ and the width δ (normalized to λ_D) are given by

$$\varphi_m^{(1)} = 3u_0/a, \qquad \delta = \sqrt{4b/u_0}. \tag{11}$$

It is found that either compressive ($\varphi_m^{(1)} > 0$) or rarefactive ($\varphi_m^{(1)} < 1$) solitary waves may exist depending on whether a is positive or negative. As β is always less than 1, it is obvious that there exists compressive solitary waves when $S_\alpha > 0$ and rarefactive solitary waves when $S_\alpha < 0$. It is seen that there exists compressive (rarefactive) solitary waves when $\alpha < (>)\, 0.155$. It is also obvious that as we increase u_0, the amplitude of these solitary structures increases whereas their width decreases.

5 Time-dependent solitary structures

We now investigate the propagation of an initial perturbation by a numerical solution of our original set of fluid equations (Eqs. (1–3)). In this numerical simulation we have used the Lax-Wendroff difference scheme where time difference Δt and space difference Δx are 0.05 and 0.10, respectively. The two initial pulses, which are used in this numerical simulation, are of the form

$$\varphi(x,0) = \begin{cases} \pm 0.025\ \mathrm{sech}^2(x-20), \\ \pm 0.025 \exp[-(x/2-10)^2]. \end{cases} \tag{12}$$

Though these initial pulses are not stationary solutions, we use the following stationary solutions for the other initial quantities:

$$u(x,0) = V_0[1-(1-2\varphi/V_0^2)^{1/2}], \tag{13}$$

$$n(x,0) = (1-2\varphi/V_0^2)^{-1/2}, \tag{14}$$

where V_0 is the velocity of the initial pulse. The value of this V_0 is chosen to be 0.773 for $\alpha = 0$ (which is nearly equal to the velocity of the solitary wave of amplitude 0.025 for $\alpha = 0$) and 0.975 for $\alpha = 0.2$ (which is close to the velocity of the solitary wave of amplitude 0.025 for $\alpha = 0.2$). We now study the time evolution of the solitary structures by solving numerically our original set of fluid equations (Eqs. (1–3)), with the initial pulses given by (12), for Maxwellian ($\alpha = 0$) and nonthermal ($\alpha = 0.2$) distribution of electrons. The numerical results are displayed in Figures 1–6. The numerical simulation in Figures 1 and 2 considers the Maxwellian electron distribution ($\alpha = 0$) and shows how two different types of initial perturbation, *viz.* $0.025\,\mathrm{sech}^2(x-10)$ and $0.025\exp[-(x/2-10)^2]$, evolve with time, whereas the numerical analyses in Figures 3–6 assume the nonthermal electron distribution ($\alpha = 0.2$) and illustrate how these two types of initial pulse, with their positive and negative forms, evolve with time. The behaviour in Figures 1–3 and 5 seems typical of the compressive solitary waves. The initial disturbance breaks up into a series of solitary waves with the largest in front. The behaviour of the rarefactive solitary waves shown in Figures 4 and 6 is quite different. These waves appear to be unstable and produce compressive waves at a later time. This behaviour, of course, rather casts doubt on our suggestion that such waves are observed, though it is possible that three dimensional structures could be more stable. This is a question which we have not examined.

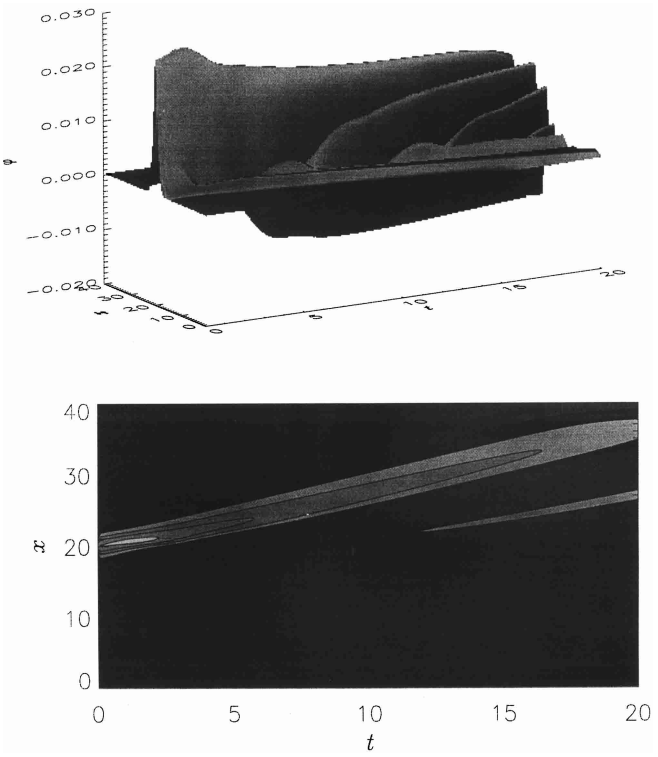

Fig. 1. Time evolution of the initial perturbation, $0.025\times$ $\mathrm{sech}^2(x-20)$, for $\alpha = 0$ and $V_0 = 0.773$. The lower view is the contour map of the upper plot.

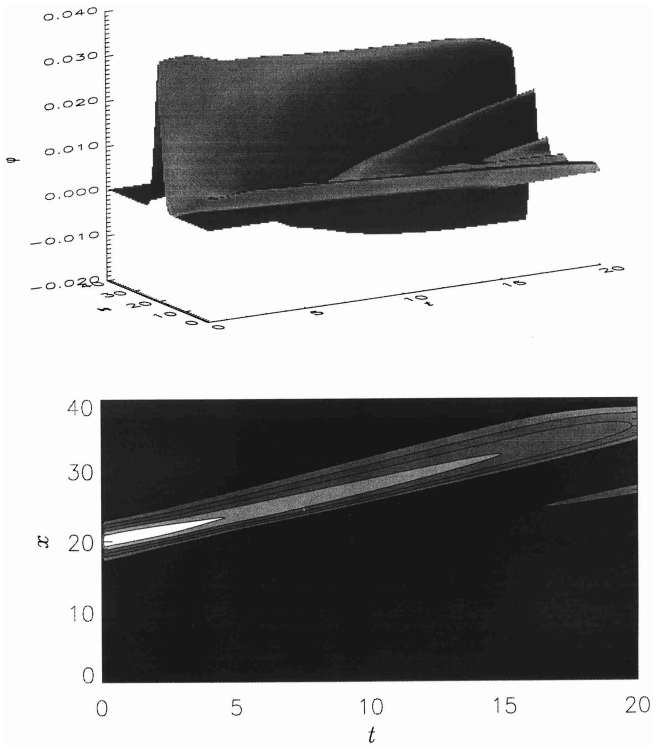

Fig. 2. Time evolution of the initial perturbation, $0.025\times$ $\exp[-(x/2-10)^2]$, for $\alpha = 0$ and $V_0 = 0.773$. The lower view is the contour map of the upper plot.

6 Discussion

Electrostatic ion-acoustic solitary structures (in a nonthermal plasma consisting of inertial ion fluid and nonthermally distributed electrons) has been investigated by pseudo-potential approach (which is valid for arbitrary amplitude waves), reductive perturbation method (which is valid for small but finite amplitude limit) and numerical

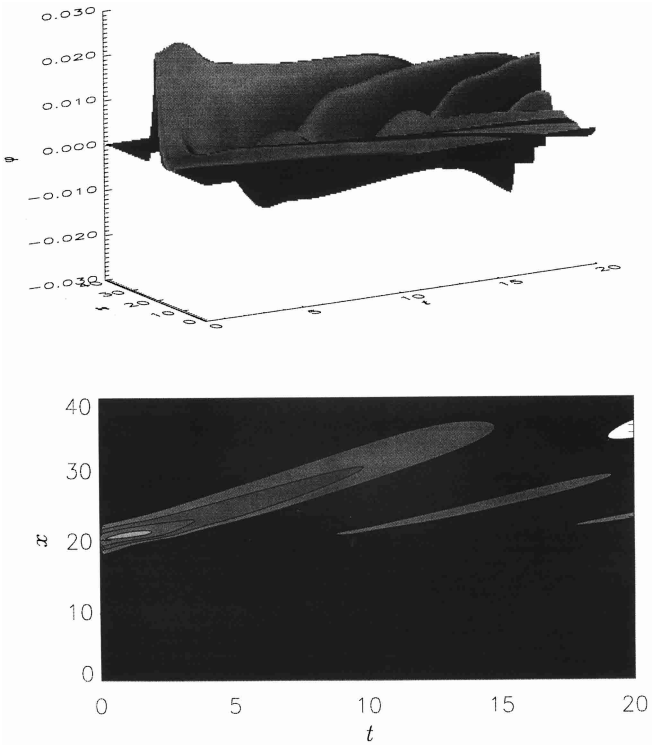

Fig. 3. Time evolution of the initial perturbation, $0.025\times \mathrm{sech}^2(x-20)$, for $\alpha = 0.2$ and $V_0 = 0.975$. The lower view is the contour map of the upper plot.

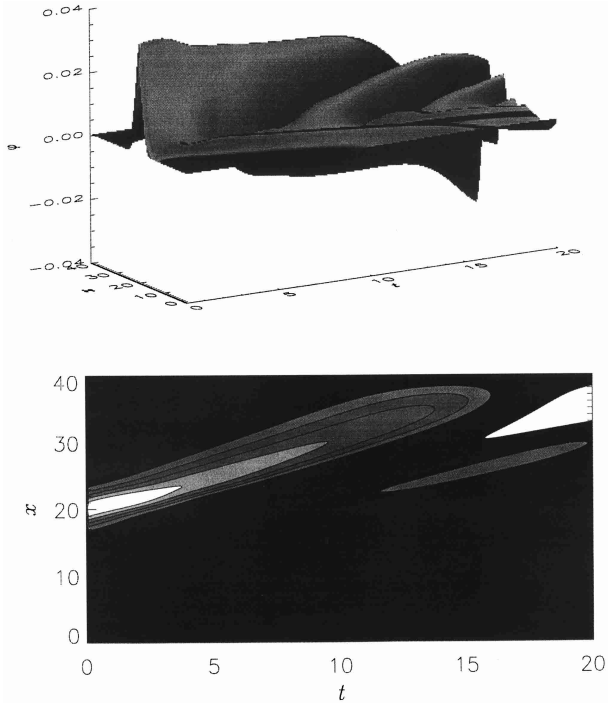

Fig. 5. Time evolution of the initial perturbation, $0.025\times \exp[-(x/2-10)^2]$, for $\alpha = 0.2$ and $V_0 = 0.975$. The lower view is the contour map of the upper plot.

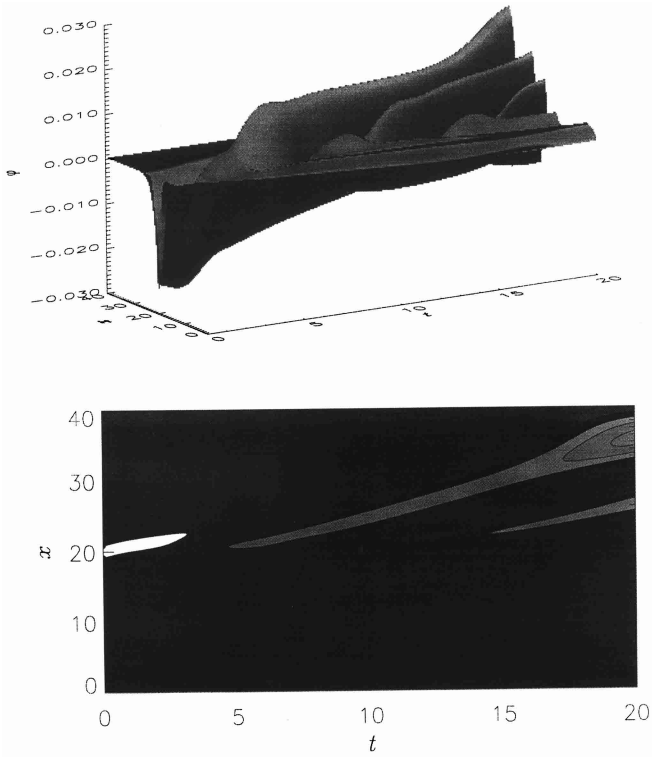

Fig. 4. Time evolution of the initial perturbation, $-0.025\times \mathrm{sech}^2(x-20)$, for $\alpha = 0.2$ and $V_0 = 0.975$. The lower view is the contour map of the upper plot.

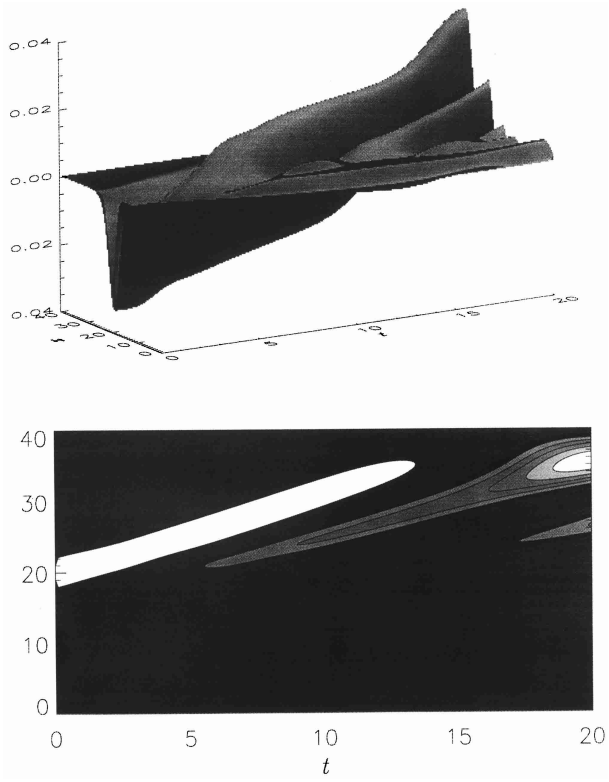

Fig. 6. Time evolution of the initial perturbation, $-0.025\times \exp[-(x/2-10)^2]$, for $\alpha = 0.2$ and $V_0 = 0.975$. The lower view is the contour map of the upper plot.

analysis (solving numerically he full set o fluid equations). The results which have been found in this investigation can be summarized as follows.

(i) The presence of the population of nonthermal (energetic) electrons modifies the nature of ion-acoustic solitary structures and allows the coexistence of compressive and rarefactive solitary waves. It is found that for $\alpha > 0.155$ compressive and rarefactive solitary waves exist together.

(ii) It is shown that as we increase α, the minimum value of the Mach number (soliton velocity normalized to ion-acoustic speed) for which rarefactive solitary waves exit, increases.

(iii) To compare the results obtained by the reductive perturbation method with those by pseudo-potential method it is found that in small amplitude limit either compressive or rarefactive solitary waves are found to exist whereas in complete theory they may exist together. In the small amplitude limit both the analyses give the same results.

(iv) It is found that as we increase the Mach number, the amplitude of both the compressive and rarefactive solitary waves increases whereas their width decreases. It is also shown that as we increase the value of α, the amplitude of the positive solitary waves increases whereas that of negative solitary waves decreases. However the width of both types of solitary structures increases with increasing the value of the parameter α.

(v) The behaviour of time-dependent positive solitary structures seems to be typical. The initial positive disturbance breaks up into a series of solitary waves with the largest in front. However, the behaviour of the rarefactive solitary waves is quite different. These waves appear to be unstable and produce compressive waves at a later time.

This analysis may be of relevance to observations in the magnetosphere of density depressions. A possible scenario is that lower hybrid turbulence produces, through modulational instability, cavities which collapse until the lower hybrid wave amplitude is sufficient to trap and accelerate a substantial number of electrons [14, 15]. The damping of the turbulence could then leave a cavity and also create just the kind of energetic electron population necessary for it to live on as an ion-acoustic solitary structure no longer supported by the ponderomotive pressure of the high frequency turbulence. However, the type of electron distribution we have looked at is common to many space and laboratory plasmas in which wave damping produces an electron tail, so the theory may be of more general interest.

It may be added here that effects of ion-temperature, external magnetic field and obliqueness on these time-dependent ion-acoustic solitary structures are also problems of current interest, but beyond the scope of the present analysis.

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